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羅必達法則

(1) 求極限 $\lim_{x\to\infty} x^{\frac{1}{x}}$ $\quad \infty^0$

原式 $= \lim_{x\to\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x\to\infty} e^{\frac{1}{x}\ln x} = e^{\lim_{x\to\infty}\frac{1}{x}\ln x}$ $\quad 0\cdot(-\infty)$

$= e^{\lim_{x\to\infty}\frac{\ln x}{x}} \; \left(\frac{\infty}{\infty}\right) \overset{L}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{x}}{1}} = e^0 = 1$

(2) 求極限 $\lim_{x\to 1^+}\left(\frac{1}{x-1} - \frac{1}{\ln x}\right)$ $\quad \infty-\infty$

原式 $= \lim_{x\to 1^+}\left(\frac{\ln x - x + 1}{(x-1)\cdot\ln x}\right)$ $\quad \frac{0}{0}$

$\overset{L}{=} \lim_{x\to 1^+}\frac{\frac{1}{x}-1}{1\cdot\ln x + (x-1)\frac{1}{x}}$ $\quad \frac{0}{0}$

$= \lim_{x\to 1^+}\frac{1-x}{x\ln x + x - 1} \overset{L}{=} \lim_{x\to 1^+}\frac{-1}{1\cdot\ln x + x\cdot\frac{1}{x} + 1}$

$= \frac{-1}{0+1+1} = -\frac{1}{2}$

$e^{-\frac{1}{0}} = e^{-\infty} = \frac{1}{e^\infty} \to 0$

(3) $\lim_{x\to 0}\frac{e^{-\frac{1}{x^2}}}{x^4} = ?$ $\quad \frac{0}{0}$

令 $\frac{1}{x^2} = t$ 　當 $x\to 0$，$t\to\infty$

原式 $= \lim_{t\to\infty}\frac{e^{-t}}{\frac{1}{t^2}} = \lim_{t\to\infty} e^{-t}\cdot t^2$ $\quad 0\cdot\infty$

$= \lim_{t\to\infty}\frac{t^2}{e^t} \overset{L}{=} \lim_{t\to\infty}\frac{2t}{e^t} \overset{L}{=} \lim_{t\to\infty}\frac{2}{e^t} = 0$

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