

微分的技巧

基本公式

$$(1) \text{乘法: } (f \cdot g)' = f' \cdot g + f \cdot g'$$

$$(2) \text{除法: } \left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$(3) \text{連鎖律: } [f(g)]' = f'(g) \cdot g'$$

$$\text{函數: 多項式: } f(x) = x^n \quad f'(x) = nx^{n-1} \quad \}$$

指數、對數:

$$\left. \begin{aligned} \underline{f(x) = e^x} &\Rightarrow f'(x) = e^x, & \underline{f(x) = \ln x} &\Rightarrow f'(x) = \frac{1}{x} \\ e^{\ln x} &= \underline{x} \quad x > 0 & \ln e^x &= x \quad x \in \mathbb{R} \end{aligned} \right\}$$

三角函數

$$\left. \begin{aligned} f(x) = \sin x &\Rightarrow f'(x) = \cos x \\ f(x) = \cos x &\Rightarrow f'(x) = -\sin x \\ f(x) = \tan x &\Rightarrow f'(x) = \sec^2 x \\ f(x) = \sec x &\Rightarrow f'(x) = \sec x \tan x \end{aligned} \right\}$$

反三角

$$\left. \begin{aligned} f(x) = \sin^{-1} x &\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}} \\ * f(x) = \tan^{-1} x &\Rightarrow f'(x) = \frac{1}{1+x^2} \end{aligned} \right\}$$

$$\sin(\sin^{-1} x) = x \quad |x| < 1$$

$$\sin^{-1}(\sin x) = x \quad |x| < \frac{\pi}{2}$$