

連續函數

假設

$$f(x) = \begin{cases} -1, & \text{若 } x \leq 0 \\ \alpha x + \beta, & \text{若 } 0 < x < 1 \\ 1, & \text{若 } x \geq 1 \end{cases} \quad \text{定義如左式}$$

若 $f(x)$ 為連續函數，試求 α 及 β 之值

$$\begin{aligned} \text{Sol. } \lim_{x \rightarrow 0^-} f(x) &= -1 \\ \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} (\alpha x + \beta) = \beta \end{aligned} \quad \left. \vphantom{\lim_{x \rightarrow 0^-} f(x)} \right\} \beta = -1$$

$$\begin{aligned} \lim_{x \rightarrow 1^-} f(x) &= \lim_{x \rightarrow 1^-} (\alpha x + (-1)) = \alpha - 1 \\ \lim_{x \rightarrow 1^+} f(x) &= 1 \end{aligned} \quad \left. \vphantom{\lim_{x \rightarrow 1^-} f(x)} \right\}$$

$$\alpha - 1 = 1 \quad \therefore \alpha = 2$$

$$\alpha = 2, \beta = -1$$