

夾擠原理求極限值

試求下列極限值 (若極限存在)

$$\lim_{x \rightarrow \infty} \left(\frac{1}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+2}} + \dots + \frac{1}{\sqrt{x^2+x}} \right)$$

$$\begin{aligned} \text{Sol. } & \left(\frac{1}{\sqrt{x^2+x}} + \frac{1}{\sqrt{x^2+x}} + \dots + \frac{1}{\sqrt{x^2+x}} \right) \leftarrow \textcircled{+} \\ & \leq \left(\frac{1}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+2}} + \dots + \frac{1}{\sqrt{x^2+x}} \right) = \text{原式} \\ & \leq \left(\frac{1}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+1}} + \dots + \frac{1}{\sqrt{x^2+1}} \right) \leftarrow \text{下} \end{aligned}$$

$$\lim_{x \rightarrow \infty} \frac{\textcircled{x}}{\sqrt{x^2+x}} = 1 \leftarrow$$

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$$\therefore \lim_{x \rightarrow \infty} \left(\frac{1}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+2}} + \dots + \frac{1}{\sqrt{x^2+x}} \right) = 1$$