換底公式

$$\log_a b = \frac{\log_c b}{\log_c a}$$ c 自定 $c>0, c\neq 1$

(1) $(\log_a b)(\log_b c)(\log_c d)\cdots\cdots(\log_y z) = \log_a z$

$\frac{\log b}{\log a}\cdot\frac{\log c}{\log b}\cdot\frac{\log d}{\log c}\cdots\cdots\frac{\log z}{\log y} = \frac{\log z}{\log a} = \log_a z$

$(\log_2 7)(\log_7 3)(\log_3 8) = \log_2 8 = 3$

(2) $\log_{a^m} b^n = \frac{n}{m}\log_a b$

$\frac{\log b^n}{\log a^m} = \frac{n[\log b]}{m[\log a]} = \frac{n}{m}\log_a b$

$\log_8 16 = \log_{2^3} 2^4 = \frac{4}{3}\log_2 2 = \frac{4}{3}$

a b c

(3) 比較 $\log_{0.2} 0.5$, $\log_2 5$, $\log_{20} 50$ 之大小

(a) $\log_{0.2} 0.5 = \frac{\log\frac{5}{10}}{\log\frac{2}{10}} = \frac{\log 5 - 1}{\log 2 - 1}$ (b) $\log_2 5 = \frac{\log 5}{\log 2}$

(c) $\log_{20} 50 = \frac{\log(5\cdot 10)}{\log(2\cdot 10)} = \frac{\log 5 + 1}{\log 2 + 1}$

$\frac{\log 5 - 1}{\log 2 - 1} > \frac{\log 5}{\log 2} > \frac{\log 5 + 1}{\log 2 + 1}$