

積化和差

$$\text{公式: (1)} \quad 2 \sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$

$$(2) \quad -2 \sin \alpha \sin \beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$$

$$(3) \quad 2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$

$$(1) \text{ 試求 } \underbrace{\sin 10^\circ \sin 50^\circ \sin 70^\circ}_{\text{原式}} =$$

$$\text{Sol. 原式} = -\frac{1}{2} (\underbrace{\cos 60^\circ - \cos 40^\circ}_{\text{原式}}) \sin 70^\circ (-+)$$

$$= -\frac{1}{2} \left(\frac{1}{2} \sin 70^\circ - \underbrace{\cos 40^\circ \sin 70^\circ}_{\text{原式}} \right)$$

$$= -\frac{1}{2} \left[\frac{1}{2} \sin 70^\circ - \frac{1}{2} (\sin 110^\circ + \sin 30^\circ) \right]$$

$$= -\frac{1}{2} \left[\frac{1}{2} \sin 70^\circ - \frac{1}{2} \sin 110^\circ - \frac{1}{4} \right] = \frac{1}{8}$$

$$(2) \text{ 試求 } \underbrace{\cos 10^\circ \cos 50^\circ \cos 70^\circ}_{\text{原式}} =$$

$$\text{Sol. 原式} = \frac{1}{2} (\underbrace{\cos 60^\circ + \cos 40^\circ}_{\text{原式}}) \cos 70^\circ$$

$$= \frac{1}{2} \left(\frac{1}{2} \cos 70^\circ + \underbrace{\cos 40^\circ \cos 70^\circ}_{\text{原式}} \right)$$

$$= \frac{1}{2} \left[\frac{1}{2} \cos 70^\circ + \frac{1}{2} (\cos 110^\circ + \cos 30^\circ) \right]$$

$$= \frac{1}{2} \left[\frac{1}{2} \cos 70^\circ + \frac{1}{2} \cos 110^\circ + \frac{\sqrt{3}}{4} \right]$$

$$= \frac{\sqrt{3}}{8}$$