

半角公式及平方化倍角

公式
 $\rightarrow \sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$ (判斷)

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$(-, +)$ | $I(+, +)$

(1) $\sin \frac{\pi}{8} = ?$

$$\sin \frac{\pi}{8} = + \sqrt{\frac{1 - \cos \frac{\pi}{4}}{2}} = + \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = + \sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= \frac{\sqrt{2 - \sqrt{2}}}{2} \text{ (取+)}$$

(2) 已知 θ 為第三象限角，且 $\sin \theta = -\frac{3}{5}$ ，則

$$\sin \frac{\theta}{2} = ?$$

$$\cos \theta = -\frac{4}{5}$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} = \pm \sqrt{\frac{1 - (-\frac{4}{5})}{2}} = \pm \sqrt{\frac{\frac{9}{5}}{2}}$$

$$= \pm \sqrt{\frac{9}{10}} \text{ (取+)} = \sqrt{\frac{9}{10}}$$

$$\begin{aligned} 180^\circ < \theta < 270^\circ \\ 90^\circ < \frac{\theta}{2} < 135^\circ \end{aligned}$$

(3) 試以平方化倍角公式：

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \text{ "平方化倍角"}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

求不定積分

$$\int \sin^2 \theta d\theta = \int \frac{1 - \cos 2\theta}{2} d\theta = \frac{1}{2} \int (1 - \cos 2\theta) d\theta$$

$$= \frac{1}{2} (\theta - \frac{1}{2} \sin 2\theta) + C = \frac{\theta}{2} - \frac{1}{4} \sin 2\theta + C$$