

三角函數的恆等式

(1) 已知 $\sin \theta + \cos \theta = \frac{1}{4}$, 則

(a) $\sin \theta \cos \theta = ?$

(b) $\tan \theta + \cot \theta = ?$

Sol. $(\sin \theta + \cos \theta)^2 = \left(\frac{1}{4}\right)^2$ $\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = \frac{1}{16}$

$$2 \sin \theta \cos \theta = \frac{1}{16} - 1 = \frac{-15}{16} \quad \sin \theta \cos \theta = \frac{-15}{32} \quad (a)$$

$$\begin{aligned} \tan \theta + \cot \theta &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\frac{-15}{32}} \\ &= -\frac{32}{15} \end{aligned}$$

(2) 若 $\cos^2 \theta + 2 \cos \theta - 3 = 0$, 則 $\cos \theta = ?$

$$\begin{array}{cc} \cos \theta & 3 \\ & \times \\ \cos \theta & -1 \end{array}$$

$$(\cos \theta + 3)(\cos \theta - 1) = 0$$

$$\cos \theta = -3 \quad \text{或} \quad \cos \theta = 1$$

不合